K25P 1898

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (C.B.C.S.S.-OBE – Reg./Supple./Imp.) Examination, April 2025
(2023 and 2024 Admissions)
MATHEMATICS
MSMAT02C07/MSMAF02C07 : Measure Theory

Time : 3 Hours   Max. Marks : 80

PART – A

Answer **any 5** questions from this Part. **Each** question carries **4** marks.

1. Show that outer measure is translation invariant.
2. Show that if f is a non-measurable function then $f = 0$ a.e if and only if $\int f dx = 0$.
3. Show that every algebra is a ring and every σ – algebra is a σ – ring but that the converse is not true.
4. If c is a real number and f, g measurable functions then prove that $f + c$, cf, $f + g$, $g - f$, fg are also measurable.
5. Let $f, g \in L^p(\mu)$ and a, b be constants then show that $af + bg \in L^p(\mu)$.
6. Prove that $||f + g||_\infty \leq ||f||_\infty + ||g||_\infty$. **(5×4=20)**

PART – B

Answer **any 3** questions from this Part. **Each** question carries **7** marks.

7. Prove that the outer measure of an interval equals its length.
8. Let f_n be a sequence of integrable functions such that $\sum_{n=1}^{\infty} \int |f_n| dx < \infty$ then prove that the series $\sum_{n=1}^{\infty} f_n(x)$ converges a.e. and its sum $f(x)$ is integrable and $\int f dx = \sum_{n=1}^{\infty} \int f_n dx$.

P.T.O.

9. Show that $\int_{n=1}^{\infty} \frac{dx}{x} = \infty$.

10. Explain complete measure and σ – finite measure and give an example for each.

11. State and prove Holder's inequality. **(3×7=21)**

PART – C

Answer **any 3** questions from this Part. **Each** question carries **13** marks.

12. a) Define a Lebesgue measurable set and prove that if $m^* (E) = 0$ then E is measurable.

    b) Prove that the class of Lebesgue measurable set is a σ – algebra.

13. Suppose that f is any extended real valued function which for every x and y satisfies $f(x) + f(y) = f(x + y)$.
    Then show that

    a) f is either everywhere finite or everywhere infinite.

    b) If f is measurable and finite then $f(x) = xf(1)$ for each x.

14. State and prove Fatou's Lemma.

15. Let μ^* be an outer measure on $\mathscr{H}(\mathscr{R})$ and let S^* denotes the class of μ^*– measurable sets. Then show that S^* is a σ– ring and μ^* restricted to S^* is a complete measure.

16. a) State Minkowski's inequality.

    b) If $1 \leq p < \infty$ and $\{f_n\}$ is a sequence in $L^p(\mu)$ such that $||f_n - f_m||_p \to 0$ as n, m $\to \infty$ then prove that there exists a function f and a subsequence $\{n_i\}$ such that $\lim f_{n_i} = f$ a.e. Also show that $f \in L^p(\mu)$ and $\lim ||f_n - f_m||_p = 0$.

(3×13=39)